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The configuration of uniform sets of points in the projective plane

Tran Thi Anh Phuong, Le Thi Thu Vi, Hoang Thi Nhu Y

Department of Mathematics, University of Education, Hue University

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ABSTRACT

Given a finite set $\mathbb{X}$ of s distinct points in the projective space $\mathbb{P}^n$, we are interested in studying its uniformity property. When $n = 2$ and $\mathbb{X}$ is (i, j)-uniform, we provide a lower bound for the number s and establish a sufficient condition for $\mathbb{X}$ to lie on a line.

1. INTRODUCTION

Let $\mathbb{K}$ be a field of characteristic zero and let $\mathbb{X} = \{p_1, \ldots, p_s\}$ be a finite set of points in the projective space $\mathbb{P}^n$ over $\mathbb{K}$. The study of the geometric properties of the set $\mathbb{X}$ is a topical subject with numerous connections to problems in projective geometry. The uniformity property not only provides a geometric characterization of $\mathbb{X}$ but also can be linked to the minimum distance of linear codes in coding theory. Here, for $0 \leq i \leq s$ and $0 \leq j \leq r_{\mathbb{X}}$, where $r_{\mathbb{X}}$ is the regularity index of $\mathbb{X}$, the set $\mathbb{X}$ is called *(i, j)-uniform* if every hypersurface of degree j passing through $s - i$ points of $\mathbb{X}$ must pass through the i remaining points of $\mathbb{X}$. Many results regarding uniformity have been discovered and widely developed (Eisenbud et al., 1996; Geramita & Kreuzer, 2013). Several algebraic invariants associated to $\mathbb{X}$ have been used to characterize the uniformity property of $\mathbb{X}$. For instance, Kreuzer (2000) used the canonical module to investigate the uniformity property of $\mathbb{X}$, Linh and Long (Kreuzer et al., 2019a; Linh & Long, 2022) provided some characterizations of uniformity property of $\mathbb{X}$ in terms of the Dedekind different, and Geramita (Geramita & Kreuzer, 2013) described the uniformity property of $\mathbb{X}$ when $\mathbb{X}$ is a complete intersection.

In view of computational aspects, Migliore and Peterson (2004) provided a characterization and an algorithm for checking the uniformity via the Chow form, while Linh and Long (2022) gave an effective algorithm to test this property of $\mathbb{X}$ based on linear algebra. Unfortunately, finding an efficient algorithm for checking this property for $\mathbb{X}$ (especially, in the general case of a 0-dimensional scheme) is still an open problem.

Another approach to looking at the uniformity property of $\mathbb{X}$ is to search algebraic curves of the smallest degree that pass through all points of $\mathbb{X}$. Bastianelli, Cortini and De Poi (2014) described some geometric configurations of $\mathbb{X}$ having the Cayley–Bacharach property of degree r (i.e, having the $(1,r)$-uniform property). This work recently was generalized by Levinson and Ullery (2022) by introducing the plane configurations in the projective space.

Based on these results, we focus on investigating the possible configurations of the set $\mathbb{X}$ that satisfy the (i, j)-uniformity property. In particular, we would like to examine some cases of the following natural question (Levinson & Ullery, 2022) that states: *"If $\mathbb{X}$ is (i, j)-uniform and the number of its points does not exceed $(d + 1)\, j + i$, does it necessarily lie on a plane configuration of dimension d?"*. In this paper, we give a positive

Corresponding author: Tran Thi Anh Phuong;
E-mail: ttaphuong@dhsphue.edu.vn
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answer for the question when $n = 2$ and $d = 1$.

The paper is organized as follows. In Section 2, we recall some basic concepts including the Hilbert function, the Cayley-Bacharach property, plane configurations, and the uniformity property. We also collect some properties about these notions, especially, Theorem 2.4 tells us that if $\mathbb{X}$ has the Cayley-Bacharach property of degree r and $|\mathbb{X}| < 2r + 1$ then all its points are collinear. Section 3 is devoted to presenting our main results, in which we first prove some uniformity of the complement set in $\mathbb{X}$ by a plane configuration (see Proposition 3.1), and then we provide a geometrical configuration of $\mathbb{X}$ satisfying the *(i, j)*-uniform property (see Theorem 3.2).

2. BASIC CONCEPTS AND PROPERTIES

In the following, let $\mathbb{K}$ be a field of characteristic zero, let $P = \mathbb{K}[x_0,\dots,x_n]$ be the standard graded polynomial ring with $\deg(x_0) = \cdots = \deg(x_n) = 1$, and let $\mathbb{X} = \{p_1,\dots,p_s\}$ be a finite set of distinct points in the n-dimensional projective space $\mathbb{P}^n$ over $\mathbb{K}$. The homogeneous vanishing ideal $I_{\mathbb{X}}$ of $\mathbb{X}$ is defined by

$$I_{\mathbb{X}} = \langle f \in P \mid f \text{ homogeneous}, f(pi) = 0, \forall i = 1,\dots,s\rangle \subseteq P.$$

Then, the homogeneous coordinate ring of $\mathbb{X}$ is the residue class ring $R = P/I_{\mathbb{X}}$. This ring $R = \oplus_{i\in\mathbb{Z}} R_i$ is a graded ring of dimension 1. We recall the definition of the Hilbert function of $\mathbb{X}$, as follows.

Definition 2.1. The **Hilbert function** of $\mathbb{X}$ is defined by

$$\mathrm{HF}_{\mathbb{X}} : \mathbb{Z} \to \mathbb{Z},\ i \mapsto \dim_K(R_i).$$

Remark 2.2. The Hilbert function of $\mathbb{X}$ satisfies $\mathrm{HF}_{\mathbb{X}}(i) = 0$ if $i < 0$ and $\mathrm{HF}_{\mathbb{X}}(0) = 1$ and

$$\mathrm{HF}_{\mathbb{X}}(0) < \mathrm{HF}_{\mathbb{X}}(1) < \cdots < \mathrm{HF}_{\mathbb{X}}(r_{\mathbb{X}-1}) < \mathrm{HF}_{\mathbb{X}}(r_{\mathbb{X}}) = \mathrm{HF}_{\mathbb{X}}(r_{\mathbb{X}+1}) = s = \cdots$$

where $r_{\mathbb{X}} \geq 0$ is known as the **regularity index** of $\mathbb{X}$.

From (Kreuzer et al., 2019b; Definition 3.10), we have the following geometrical property of $\mathbb{X}$.

Definition 2.3. For $r \geq 0$, we say that $\mathbb{X}$ has the **Cayley - Bacharach property of degree** r (**CBP**(r)) if every hypersurface of degree r passing through $s - 1$ points of $\mathbb{X}$ also passes through the remaining point.

Note that if $s > 1$ then $\mathbb{X}$ has CBP(0) and if $\mathbb{X}$ has CBP(r) then it has CBP($r - 1$). Moreover, the number $r_{\mathbb{X}} - 1$ is the largest degree $r \geq 0$ such that $\mathbb{X}$ can have CBP(r) (Kreuzer et al., 2019a; Section 4]). In particular, (Bastianelli et al., 2014; Lemma 2.4) provides a geometric characterization of the set $\mathbb{X}$ satisfying the Cayley-Bacharach property, stated below.

Theorem 2.4. *Suppose that the set* $\mathbb{X} = \{p_1,\dots, p_s\} \subseteq \mathbb{P}^n$ has CBP(r). Then

(a) $s \geq r + 2$; *moreover,*

(b) *if* $s \leq 2r + 1$ then $\mathbb{X}$ lies on the line.

Concerning the study of geometric structure of $\mathbb{X}$, the notion of the plane configurations was introduced in (Levinson & Ullery, 2022; Definition 2.1).

Definition 2.5. Let d, ℓ be non-negative integers. A **plane configuration** $\mathscr{P}$ **of dimension** d and length ℓ in $\mathbb{P}^n$ is a union of distinct positive dimensional linear spaces $\mathscr{P} = \cup_{i=1}^{k} P_i$, in which:

(a) $d = \dim(\mathscr{P}) = \sum_{i=1}^{k} \dim(P_i)$ is the dimension of the plane configuration $\mathscr{P}$;

(b) $\ell(\mathscr{P}) = k$ is the length of the plane configuration $\mathscr{P}$.

Remark 2.6. (a) A zero-dimensional plane configuration is an empty set, and a 1-dimensional plane con-figuration of length 1 is a line.

(b) In the projective plane, any plane configuration $\mathscr{P}$ is either a finite union of lines or the whole plane.

Based on the above terminology, the following conjecture is proposed by Levinson and Ullery (2022), which extends Theorem 2.4.

Conjecture 2.7. (Levinson & Ullery, 2022; Conjecture 1.2). *Let* $\mathbb{X} = \{p_1,\dots, p_s\} \subseteq \mathbb{P}^n$ *be a set of s distinct points having CBP(r). If* $s \leq (d + 1)r + 1$ *then* $\mathbb{X}$ *lies on a plane configuration* $\mathscr{P}$ *of dimension* d.

Conjecture 2.7 holds in the following cases (Levinson & Ullery, 2022; Theorem 1.3):

(a) For $r \leq 2$ and all d. Moreover, we may take $\ell(\mathscr{P}) = 1$.

(b) For all r and $d \leq 3$. Moreover, for $r \leq 4$ we may take $\ell(\mathscr{P}) \leq 2$.

(c) For $r = 3$ and $d = 4$. Moreover, we may take $\ell(\mathscr{P}) \leq 2$.

The Cayley-Bacharach property of $\mathbb{X}$ is known as a weak uniform property of $\mathbb{X}$. In the next definition we recall from (Geramita & Kreuzer, 2013; Definition 2.1) the notion of uniformity of $\mathbb{X}$. This notion plays an important role in this paper.

Definition 2.8. Let $\mathbb{X} = \{p_1, \ldots, p_s\} \subset \mathbb{P}^n$ be a set of s distinct points, and let $1 \le i \le s$ and $1 \le j \le r_{\mathbb{X}}$. The set $\mathbb{X}$ is said to be (i, j)-**uniform** if and only if $\mathrm{HF}_{\mathbb{Y}}(j) = \mathrm{HF}_{\mathbb{X}}(j)$, for all subsets $\mathbb{Y} \subset \mathbb{X}$ consisting of $|\mathbb{Y}| = s - i$ points.

Remark 2.9. Notice that the set $\mathbb{X}$ is (i, j)-uniform if any degree j algebraic curve passing through $s - \mathrm{i}$ points of $\mathbb{X}$ must pass through remaining i points of $\mathbb{X}$. Indeed, let $\mathbb{Y}$ be a subset of $s - \mathrm{i}$ points of $\mathbb{X}$ and $f \in P_j$ vanishes at all points of $\mathbb{Y}$. Clearly, $I_{\mathbb{X}} \subseteq I_{\mathbb{Y}}$ and $f \in (I_{\mathbb{Y}})_j$. Because $\mathbb{X}$ is (i, j)-uniform, we have $\mathrm{HF}_{\mathbb{Y}}(j) = \mathrm{HF}_{\mathbb{X}}(j)$, and so $(I_{\mathbb{X}})_j = (I_{\mathbb{Y}})_j$. It follows that $f \in (I_{\mathbb{X}})_j$, in other words, f vanishes at all i remaining points of $\mathbb{X}$.

In particular, due to Definition 2.3, $\mathbb{X}$ has CBP(r) if and only if it is $(1, r)$-uniform.

Example 2.10. Let $\mathbb{X} = \{p_1, \ldots, p_s\}$ be the set of 10 points in the projective space $\mathbb{P}^2$ over $\mathbb{Q}$, where $p_1 = (1:0:2)$, $p_2 = (1:1:0)$, $p_3 = (1:1:1)$, $p_4 = (1:1:2)$, $p_5 = (1:2:1)$, $p_6 = (1:2:2)$, $p_7 = (1:3:0)$, $p_8 = (1:3:1)$, $p_9 = (1:3:2)$ and $p_{10} = (1:3:3)$. Then, by using ApCoCoA (n.d.) we have the homogeneous vanishing ideal $I_{\mathbb{X}}$ of X is minimally generated by 7 homogeneous polynomials in $P = \mathbb{Q}[x_0, x_1, x_2]$:

$$
\begin{aligned}
f_1 &= x_0^2x_1x_2 - x_0^2x_2^2 - \tfrac{3}{2}x_0x_1x_2^2 + \tfrac{3}{2}x_0x_2^3 + \tfrac{1}{2}x_1x_2^3 - \tfrac{1}{2}x_2^4,\\
f_2 &= x_0^3x_2 - \tfrac{11}{6}x_0^2x_2^2 + x_0x_2^3 - \tfrac{1}{6}x_2^4,\\
f_3 &= x_0^2x_1^2 - \tfrac{4}{3}x_0x_1^3 + \tfrac{1}{3}x_1^4 - \tfrac{1}{3}x_1^3x_2 - x_0^2x_2^2 + \tfrac{23}{6}x_0x_1x_2^2 + \tfrac{2}{3}x_1^2x_2^2\\
&\quad - \tfrac{5}{2}x_0x_2^3 - \tfrac{13}{6}x_1x_2^3 + \tfrac{3}{2}x_2^4,\\
f_4 &= x_0^3x_1 - \tfrac{13}{9}x_0x_1^3 + \tfrac{4}{9}x_1^4 - \tfrac{11}{18}x_1^3x_2 - \tfrac{11}{6}x_0^2x_2^2 + \tfrac{253}{36}x_0x_1x_2^2 + \tfrac{11}{9}x_1^2x_2^2\\
&\quad - \tfrac{55}{12}x_0x_2^3 - \tfrac{143}{36}x_1x_2^3 + \tfrac{11}{4}x_2^4,\\
f_5 &= x_0^4 - \tfrac{40}{27}x_0x_1^3 + \tfrac{13}{27}x_1^4 - \tfrac{1}{2}x_0x_1^2x_2 - \tfrac{19}{27}x_1^3x_2 - \tfrac{85}{36}x_0^2x_2^2\\
&\quad + \tfrac{563}{54}x_0x_1x_2^2 + \tfrac{85}{54}x_1^2x_2^2 - \tfrac{61}{9}x_0x_2^3 - \tfrac{301}{54}x_1x_2^3 + \tfrac{47}{12}x_2^4,\\
f_6 &= x_1^4x_2 - \tfrac{39}{2}x_0x_1^2x_2^2 - 3x_1^3x_2^2 + \tfrac{111}{2}x_0x_1x_2^3 + \tfrac{25}{2}x_1^2x_2^3 - 36x_0x_2^4 - \tfrac{57}{2}x_1x_2^4 + 18x_2^5,\\
f_7 &= x_0x_1^3x_2 - 6x_0x_1^2x_2^2 - \tfrac{1}{2}x_1^3x_2^2 + 11x_0x_1x_2^3 + 3x_1^2x_2^3 - 6x_0x_2^4 - \tfrac{11}{2}x_1x_2^4 + 3x_2^5.
\end{aligned}
$$

The Hilbert function of $\mathbb{X}$ is $\mathrm{HF}_{\mathbb{X}} = (1, 3, 6, 10, 10, \ldots)$, and so the regularity index of $\mathbb{X}$ is $r_{\mathbb{X}} = 3$. In particular, we can check that $\mathbb{X}$ is (i, j)-uniform for $i = 1, 2$ and $j = 2$, but $\mathbb{X}$ is not $(3, 2)$-uniform, for instance, for $\mathbb{Y} = \{p_1, p_3, p_4, p_5, p_6, p_8, p_9\} \subseteq \mathbb{X}$ we get $|\mathbb{Y}| = 7$ and $\mathrm{HF}_{\mathbb{Y}}(2) = 5 < 6 = \mathrm{HF}_{\mathbb{X}}(2)$.

The possible region of uniformity of the set $\mathbb{X}$ can be described by the following lemma (Kreuzer, 2002; Remark 2.2).

Lemma 2.11. *Let* $\mathbb{X} = \{p_1, \ldots, p_s\} \subset \mathbb{P}^n$ *be a set of s distinct points. Suppose that* $\mathbb{X}$ is (i, j)-*uniform for some* $1 \le i \le s$ and $1 \le j < r_{\mathbb{X}}$.

(a) *We have* $i \le s - \mathrm{HF}_{\mathbb{X}}(j)$.

(b) *For all* $1 \le i' \le i$ *and* $1 \le j' \le j$, the set $\mathbb{X}$ is also (i', j')-*uniform.*

3. THE MAIN RESULT

In this section, we use the notation and definitions given in the previous section. We aim to prove a condition that determines whether a uniform set of points, $\mathbb{X} = \{p_1, \ldots, p_s\}$, in the projective plane lies on a line. Using the language of uniformity, we formulate the Levinson-Ullery conjecture 2.7 in $\mathbb{P}^2$ as follows: If a set of s distinct points $\mathbb{X} = \{p_1, \ldots, p_s\} \subseteq \mathbb{P}^2$ *satisfies* (i, j)-*uniform and if* $s \le (d+1)j + \mathrm{i}$ *then* $\mathbb{X}$ *lies on a plane configuration* $\mathscr{P}$ *of d -dimension.*

In the following, we prove this problem for the case $d = 1$. For that, we need the following result.

Proposition 3.1. *Let* $\mathbb{X} \subseteq \mathbb{P}^2$ *be a finite set of points satisfying* (i, j)-*uniform and let* $\mathscr{P}$ *be a plane configuration with length l . Then the set* $\mathbb{X} \setminus \mathscr{P}$ *satisfies* $(i, j-l)$-*uniform. In particular, if* $\mathscr{P}$ *is a line, then* $\mathbb{X} \setminus \mathscr{P}$ *is* $(i, j-1)$-*uniform.*

Proof. By remark 2.6, it suffices to prove the proposition in the case $\mathscr{P}$ is a plane configuration of length 1, i.e., $\mathscr{P}$ is a line. Setting $\overline{\mathbb{X}} = \mathbb{X} \setminus \mathscr{P}$, we have to show that $\overline{\mathbb{X}}$ is $(i, j-1)$-uniform. Let Z be a hypersurface of degree $j - 1$ passing through all points of $\overline{\mathbb{X}}$ except for i points of them, and let W be the set of these i points. Then, we have $|W| = i, W \cap \mathscr{P} = \emptyset$ and

$$\overline{\mathbb{X}} \setminus W \subseteq \mathrm{Z}.$$

Now, we consider the hypersurface $\mathscr{P} \cup \mathrm{Z}$ of degree j. This hypersurface contains both $\overline{\mathbb{X}} \setminus W$ and $\mathscr{P}$. It follows that

$$(\mathbb{X} \setminus W) \subseteq (\mathscr{P} \cup \mathrm{Z}).$$

On the other hand, since $\mathbb{X}$ is (i, j)-uniform and $|W| = i$ so $W \subseteq (\mathscr{P} \cup \mathrm{Z})$, and $W \cap \mathscr{P} = \emptyset$, we conclude $W \subseteq \mathrm{Z}$. Therefore, Z is a hypersurface of degree $j - 1$ passing through all points in both $\overline{\mathbb{X}} \setminus W$ and W , which implies that the set $\overline{\mathbb{X}}$ is $(i, j - 1)$-uniform. □

Now, we are ready to state and prove the main result of this section.

Theorem 3.2. *Let $\mathbb{X} = \{p_1, \dots, p_s\}$ be a set of s distinct points in the projective plane $\mathbb{P}^2$ and suppose that $\mathbb{X}$ is (i, j)-uniform.*

(a) We have $|\mathbb{X}| = s \geq j + i + 1$.

(b) If $s \leq 2j + i$ then $\mathbb{X}$ lies on a line.

Proof. (a) Suppose for a contradiction that $s < i + j + 1$. Since $\mathbb{X}$ is (i, j)-uniform, every algebraic curve of degree j passing through $s - i$ points of $\mathbb{X}$ also passes through the remaining i points of $\mathbb{X}$. Let us consider $s - i$ lines $\ell_1, \dots, \ell_{s-1}$ in $\mathbb{P}^2$ with $p_k \in \ell_k$ for $k = 1, \dots, s - i$ and $p_m \notin \bigcup_{k=1}^{s-i} \ell_k$ for $m = s - i + 1, \dots, s$.

Let F_{j-s+i} be an algebraic curve of degree $j - s + i$, not passing through the points $p_{s-i+1}, \dots, p_s$, with $F_{j-s+i} = \emptyset$ when $j = s - i$. In this case, we define

$$H = F_{j-s+i} \cup \ell_1 \cup \ell_2 \cup \cdots \cup \ell_{s-i}$$

Then H is an algebraic curve of degree j which does not pass through the points $p_{s-i+1}, \dots, p_s$. This contradicts the fact that $\mathbb{X}$ is (i, j)-uniform. Hence we must have $s \geq i + j + 1$.

(b) Now we suppose that $s \leq 2j + i$. We need to show that $\mathbb{X}$ lies on a line. To this end, we distinguish the following cases.

(b.1) $i = j = 1$: The set $\mathbb{X}$ is (1, 1)-uniform, and so

$$i + j + 1 \leq s \leq 2j + i \Leftrightarrow 3 \leq s \leq 3 \Leftrightarrow s = 3.$$

Thus $\mathbb{X} = \{p_1, p_2, p_3\}$ and $\mathbb{X}$ has CBP(1). It follows that $\mathbb{X}$ lies on a line.

(b.2) $i = 1$ and $j \geq 2$: We have $s \leq 2j + 1$ and $\mathbb{X}$ is $(1, j)$-uniform, that is, $\mathbb{X}$ has CBP(j). By Theorem 2.4, $\mathbb{X}$ must lie on a line.

(b.3) $j = 1$ and $i \geq 2$: We find that $s \leq 2 + i$ and $\mathbb{X}$ is $(i, 1)$-uniform. Then, every line passing through $s - i$ points also passes through the remaining i points of $\mathbb{X}$. Thus, $\mathbb{X}$ must be collinear.

(b.4) $i \geq 2$ and $j \geq 2$: In this case, we may assume that any set of t distinct points in $\mathbb{P}^2$, which is (i, k)-uniform with $1 \leq k < j$ and where $1 \leq t \leq \min\{2k + i, s\}$, also lies on a line. Consider the set $\mathbb{X}$, let h be the maximum number of collinear points in $\mathbb{X}$ lying on a line ℓ and let Y be the set consisting of these h collinear points. Clearly, $Y \subseteq \mathbb{X}$ and $|Y| = h$. Put $Z = \mathbb{X} \setminus Y$. Then

$$Z \cap \ell = \emptyset,\ Y \subseteq \ell,\ |Z| = s - h.$$

Observe that if $Z = \emptyset$ then $Y = \mathbb{X}$ lies on a line ℓ. So, it is enough to prove $Z = \emptyset$. Assume for a contradiction that $Z \neq \emptyset$. Because $Z = \mathbb{X} \setminus \ell$ and $\mathbb{X}$ is (i, j)-uniform, an application of Proposition 3.1 yields that the set Z is $(i, j - 1)$-uniform in $\mathbb{P}^2$. By part (a), we have

$$|Z| \geq i + (j - 1) + 1 = i + j.$$

This implies

$$s - h \geq i + j.$$

For $h \geq 2$, one has

$$i + j \leq s - h \leq (2j + i) - h \leq 2j + i - 2 = 2(j - 1) + i.$$

By the inductive hypothesis, the set Z lies on line ℓ' . Consequently, the set $\mathbb{X}$ is contained in $\ell \cup \ell'$. By the choice of h, we get the inequality $s - h \leq h$, and so $\frac{s}{2} \leq h$. It follows that

$$i + j \leq s - h \leq \frac{s}{2} \leq \frac{2j + i}{2} = j + \frac{i}{2}.$$

Hence

$$i + j \leq \frac{i}{2} + j,$$

which is a contradiction. Therefore, it must be $Z = \emptyset$ in this case.

Altogether, the set $\mathbb{X}$ lies on a line, and this completes the proof. □

Remark 3.3. Theorem 3.2 also covers a result of Bastianelli, Cortini and De Poi when $n = 2$ and $i = 1$ (see (Bastianelli et al., 2014; Lemma 2.4). Our future work is to generalize this theorem for $n > 2$ and consider the initial index of the homogeneous vanishing ideal of $\mathbb{X}$.

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Tính đều của tập hữu hạn điểm trong mặt phẳng xạ ảnh

Trần Thị Anh Phương, Lê Thị Thu Vi, Hoàng Thị Như Ý

Khoa Toán học, Trường Đại học Sư phạm, Đại học Huế

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Từ khóa:
Các tập điểm;
Tính Cayley–Bacharach;
Các cấu hình phẳng Tính đều.

Tác giả liên hệ:
Trần Thị Anh Phương
Địa chỉ e-mail:
ttaphuong@dhsphue.edu.vn

TÓM TẮT

Với $\mathbb{X}$ là một tập hợp hữu hạn gồm s điểm phân biệt trong không gian xạ ảnh $\mathbb{P}^n$, chúng tôi quan tâm đến việc nghiên cứu tính đều của tập điểm đó. Khi $n = 2$ và $\mathbb{X}$ là (i, j)-đều, chúng tôi đưa ra một chặn dưới cho số s và chứng minh một điều kiện đủ để $\mathbb{X}$ nằm trên một đường thẳng.